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Strategic Choice of Contract Lenghts in Agriculture

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Strategic Lengths of Tenancy Contracts *

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Abstract

In a multi-period environment, time is certainly an added dimension in which the principal can differentiate in search of better screening contracts. This idea is used in a two-period model of agriculture without production uncertainties, where the landlord offers separating tenurial contracts to screen tenants of different skills, when the skill is private information. The paper shows that a risk neutral landlord wishing to hire a risk averse tenant will always find it optimal to offer a menu consisting of a long term fixed rent contract and a short term sharecropping contract. Independent of whether effort is observable or not, self selection leads to a high skilled tenant working under the long term scheme while a low skilled tenant working under the short term one.

KEYWORDS: short-term versus long-term tenancy contracts, risk aversion, screening.

JEL Classifications: J41, J43, O12.

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1 Introduction

A situation where the owner of a physical asset (or input in general) hires an agent to manage it and produce output is very common in economics, e.g. landlord-tenant in agriculture, timber sales, gold mining, oil and gas leasing, franchise and patent licensing. The contractual arrangement by which the agent is compensated for his input(s) into the productive process determines the level of output and hence the level of efficiency. The length of such contracts (for example duration of employment) also plays a pivotal role in forming strong and lasting relationships between the involved parties and promoting efficiency. Static contract theory is primarily concerned with how the contract influences current performance ignoring the interplay between the characteristics of the contract and the incentives it provides for future effort. Dynamic contract theory fills this gap [e.g. Rubinstein and Yaari (1983), Radner (1985), Holmstrom and Milgrom (1987), Spear and Srivastava (1987) and Phelan and Townsend (1991)].

The aim of this paper is to study the nature and length of tenancy contracts in a dynamic setting of incomplete information and with imperfect capital markets. In particular, we focus on contractual agreements in agriculture which deserve a special attention. On one hand they share many common features with other contracts and hence by studying them we learn more about general contracting properties. On the other hand, they exhibit certain peculiarities which cannot be easily found in other environments. For instance, financial market imperfections are more widespread in less developed economies (where agricultural contracts are more important) than in developed ones and impose restrictions on the contracting abilities of the economic players. Also, precision and the speed of information acquisition are different in a rural society than in an advanced one.

There is empirical evidence that tenurial contracts in agriculture vary in length.[1] Bandiera (1999), for example, examines tenurial contracts from the 19th century rural Sicily, where she finds that 26.2% of them are for one year, 6.9% for two years, 10.9% for three years, 35% for four years

[1] The role of contract lengths appears to be prevalent in other economic areas as well. For example, franchise contractual agreements exhibit significant variation in length [e.g. Lafontaine (1992), table 2, p.270]. Although we study the landlord-tenant paradigm in agriculture, our approach can be applied to any dynamic principal-agent model.

and 21% for five years or more. She also notes that "*most sharecropping contracts were one year long*," that is short term. Even more interestingly, Newbery (1977) writes

> "*Almost all share contracts are for one year or one crop season, and are thus more like labour contracts than fixed rent contracts, which are typically for a long period.* "

It is exactly this type of evidence coupled with the above assertion of Newbery that we explain using a dynamic model of agriculture with *moral hazard* and *adverse selection*. In particular we provide a rationale by which long term contracts are accompanied with a fixed-rent, while short term contracts involve sharecropping.

Contrary to the literature on share tenancy in a static environment,[2] there is not much work on the issue of contract length in agriculture. Bandiera tests the hypothesis that the length of contracts depends crucially on the sensitivity of the crop to maintenance. Since maintenance effort is not contractible, a landlord with a very sensitive crop (like vines and citrus trees), offers to a tenant a long term contract in order to induce him to take the highest possible care of the crop. This incentive provision is costly because of information rents, and therefore short term contracts are used when the crop is not sensitive to maintenance effort (like wheat). This cost comes from a limited-liability assumption by which tenants are not liable to the landlord for more than their wealth. There exists also a literature[3] which argues that long term contracts are costly due to the fact that the landlord cannot use the *threat of eviction* to elicit a higher effort on the part of the tenant. A tenant operating under a short term contract will increase his effort today in order to (probabilistically) increase the output and consequently lower the probability of eviction. On the other hand, he may not have enough incentives for improvements which enhance future productivity [e.g. Bardhan (1984), Ch. 8]. Nevertheless, if the horizon is infinite [Bardhan (1984)

[2] There are many papers, for instance, which try to justify the existence of share tenancy. For an extensive list of references on this issue see Singh (1989), Basu (1997) and Ray (1998). Such issues are also addressed in a dynamic environment with differential intertemporal preferences in Roy and Serfes (2000). However, the length of contracts is not endogenously determined in their analysis.

[3] For an excellent survey on contracts with eviction see Dutta, Ray and Sengupta (1989, section 2).

considers a two-period model], then short term contracts provide incentives for land improvement since the tenant will reap the benefits of current investment in the coming years which will increase production and lower the probability of eviction.

We consider a village which consists of one landlord who owns a piece of land and has a two cropping-period (period, henceforth) planning horizon. He hires a tenant in order to carry out farm management. We assume that *the tenant is more risk averse than the landlord.*[4] Farm management entails hiring and supervision of unskilled workers which demands skill and costly effort on the part of the employed tenant. A tenant who wishes to work for the landlord is either high-skilled or low-skilled. However, at the beginning of period 1 when the landlord decides to hire a tenant, such skills are private information and the landlord has prior beliefs regarding this.[5] Output produced by the tenant is observed in both periods but the level of effort may not be so. If effort is observable, an assumption plausible under many instances,[6] true skills cannot be *misrepresented.* In that case at the end of period 1, the true skills become common knowledge. Otherwise, that is when effort remains unobserved, the tenant may misrepresent his true skill in period 1 which, in that case, may remain unknown even in period 2.

The landlord has the option of offering contracts for one or two periods. A one period contract is typically referred to as a short term contract, while a two period contract is a long term one. We concentrate on the issue of separating contracts by which the landlord wishes to screen between the

[4]There is no exogenous uncertainty in our model. A risk averse tenant is defined as the one who evaluates any outcome by using a concave utility function. The assumption that the tenant is more risk averse than the landlord can be argued to be consistent with the former being poorer than the latter. In a similar set up as ours but with complete information, Holmstrom (1983) shows that risk neutral firms offer long term contracts to risk averse workers to insure them against downside risk.

[5]One may argue that in the small world of a traditional village the landlord has a fairly informative idea about the skill of each tenant and therefore these skills are not private information. While this argument is to a certain extent valid, it does not capture the whole picture. The reasons are: 1) The above claim certainly looses its gravity in a modern society. 2) Skills do not remain fixed but change over time depending upon a tenant's (unobserved) investments in human and physical capital. For example, the quality of a tenant's tractor or animal depends on his maintenance effort, which is to a great extent private information. Also, the improvement of skills is related to a tenant's *cognitive ability*, i.e., how he evaluates his experience and how fast he learns. 3) There may be an inflow of new tenants into the village with completely unknown skills. 4) The skill may also depend on the *psychological state* of the tenant. Due to unobserved (to the landlord) events even a high skilled tenant may, in a given period of time, "feel" like working as a low skilled one. Therefore, we believe that at least part of a tenant's skill is idiosyncratic and initially it is unknown to the landlord. Our model considers the extreme case where the skill is private information when the contract is offered but potentially it can become common knowledge in the second period.

[6]Effort can also be thought of as either the number of unskilled laborers that the tenant employs or the hours of work that the tenant puts in farming, which are to a great extent observable to the landlord who is around.

two types of tenants by offering a menu of contracts over the two periods. Along with such menus he also specifies the duration over which the tenant is employed. We show the following.

1. In the event the true skill of the tenant is known to the landlord from the very beginning, the landlord is indifferent between short and long term contracts. Of course, with an uncertain future, the landlord will always prefer long term contracts if the tenants are more risk averse. Thus, the choice between different durations will be solely driven by mechanisms (at the disposal of the landlord, or socially enforced in the village) which may control the degree of enforceability of long term contracts. However, this provides only an exogenous theory for the issue at hand. Therefore, we do not model future uncertainties as our goal is to show that the choice between short term and long term contracts may be driven endogenously solely by the landlord's desire to screen between the different types of tenants and take advantage of the duration of contracts.

2. When the true skill of the tenant is private information, short and long term contracts *co-exist*. In particular, in any optimal separating scheme, the landlord always offers a long term *fixed rent* contract for the high-skilled tenant and a short term *sharecropping* contract for the low-skilled one. This is because, in any separating equilibrium, the high-skilled tenant enjoys a positive surplus in the first period, but a zero surplus in the second when his true type is revealed. Since tenants are risk averse, a risk neutral landlord can become better off by offering the high-skilled tenant a long term contract and *smoothing* his payoff.[7] However, in doing so, the landlord must be careful that the menu of contracts is intertemporally incentive compatible. For this reason, the contract offered to the low-skilled tenant is of a short duration. This prevents the high-skilled tenant from locking himself with the contracts offered to the low-skilled tenant, which he prefers more. These results are not affected when the tenant can strategically misrepresent his true skill. The exact terms of the contracts change in that case, but the durations of tenancy remain unaltered.

[7] With a fully developed financial sector one may argue that the tenant can transfer wealth between periods and therefore the income smoothing role of a long term contract can be replicated via intertemporal wealth transfer mechanisms. However an assumption of the existence of an efficient financial sector is far fetched in many rural communities. In such areas the only available savings technologies are a) lending money to some other agents, b) keeping wealth in hand, and c) using the landlord as the source of finance (interlinkages). In the development literature there is ample evidence of the riskiness of schemes a) and b). On the other hand, c) involves unnecessarily many transactions and certainly more than the mechanism of income smoothing through the use of long term contracts we propose. Moreover, even in an economy with a developed financial sector, our contract smooths the agent's consumption without him having to engage in further saving transactions which certainly involve extra "costs."

The rest of the paper is organized as follows. In section 2, we formally describe the model and discuss the benchmark case where the true type of the tenant is observable. Section 3 studies the screening problem faced by the landlord when the skill of the tenant is private information and effort is observable. Section 4 extends this to the case where effort cannot be observed and therefore, misrepresentation of true skills is possible. In section 5 we address the issue of more than two periods and more than two possible types. The paper concludes in section 6.

2 The environment

Consider a village with one landlord and one tenant who operate under a two period horizon, where each period is denoted by $t = 1, 2$. The landlord owns a fertile piece of land which can be used to grow a non-storable foodgrain in each period. Cultivation requires skill and effort. Let q_t denote the quantity of foodgrain produced in period t. For simplicity, we assume that the price of foodgrain is the same in both periods and equal to one. The landlord needs to hire the tenant to carry out cropping. Let θ denote the skill of the tenant, $\theta \in \{h, \ell\}$, with $\infty > h > \ell > 0$. We assume that the landlord cannot observe θ and let μ denote the subjective probability (held by the landlord) that $\theta = h$. The production function for foodgrain is $q_t(e_t; \theta) = \theta f(e_t)$, where $e_t \in \mathbb{R}_+$ is the level of effort that the tenant exerts in period t. We assume that $f' > 0$ and $f'' < 0$. Effort is costly and let $c(e_t; \theta) = e_t$ be the period t cost of production.[8] Suppose q_t is observable in both periods, but the landlord may or may not observe e_t.

In order to hire the tenant, the landlord offers a contract. The contracts we study belong to the class of linear contracts represented by a pair (α_t, β_t), where $\alpha_t \in [0, 1]$ is the landlord's share of total period t output and $\beta_t \in \mathbb{R}$ is a fixed payment from the tenant to the landlord in period t.[9] The landlord has two options to choose from. He may want to hire the tenant separately for each period. In that case, he announces (α_1, β_1) at the beginning of period 1. When period 2 comes, he announces (α_2, β_2). Such contracts will be referred to as *short term contracts*. He may also wish to hire the tenant (once and for all) for two periods. In that case, he announces $(\alpha_t, \beta_t)_{t=1,2}$ at the

[8] For simplicity, we have assumed such functional forms of the technology and the cost. Working with more general functions will not affect our results qualitatively. Also, one may assume e_t to denote the amount of unskilled labor which can be hired at a fixed per unit wage of 1.

[9] We do not study wage or other output contracts.

beginning of period 1. Such contracts will be referred to as *long term contracts*. We assume that once the tenant accepts a contract, the terms of the contract become binding. Thus, if the tenant accepts a long term contract, he cannot run away until his two-period term is over. We further assume that the landlord has all the bargaining power. Our analysis is based on the assumption that there is full commitment and therefore we rule out the possibility of renegotiation.

Denote by $\left(\alpha_t^\theta, \beta_t^\theta\right)$ the contract under which the tenant of skill θ works in period t. In order to separate between the two possible types of the tenant, the landlord may not only use different specifications of shares and rents, but may also offer different durations if doing so is beneficial for him. In this spirit, denote by $(t_h - t_\ell)$ the scenario where the landlord designs his separating scheme to hire the high-skilled tenant for t_h periods and the low-skilled tenant for t_ℓ periods, $t_h = 1, 2$ and $t_\ell = 1, 2$. We will refer to such a scenario as the $(t_h - t_\ell)$ scheme. The four possible schemes available to the landlord are the $(1-1)$ scheme, the $(2-1)$ scheme, the $(1-2)$ scheme and the $(2-2)$ scheme. Under each scheme, the landlord decides upon a sequence of contracts $\left\{\left(\alpha_1^h, \beta_1^h\right), \left(\alpha_1^\ell, \beta_1^\ell\right), \left(\alpha_2^h, \beta_2^h\right), \left(\alpha_2^\ell, \beta_2^\ell\right)\right\}$ over the two periods.

2.1 The two–period game form

The game played between the landlord and the tenant has the following stages.

<u>*Stage 1*</u>. At the beginning of period 1, the landlord decides upon which scheme of contracts to offer.[10]

<u>*Stage 2.*</u> The tenant decides which duration of contracting to work under given the landlord's choice in period 1. If he rejects all contracts offered, no production takes place in period 1. If he accepts a short term contract, he chooses his level of effort and production takes place in period 1. In either of these cases, we move to stage 3. Otherwise, that is if he accepts a long term contract, the game ends. In each of the two periods, he chooses his level of effort and production takes place over time.

<u>*Stage 3.*</u> We enter period 2 when the landlord offers a (possibly fresh) pair of separating contracts $\left(\alpha_2^h, \beta_2^h\right)$and $\left(\alpha_2^\ell, \beta_2^\ell\right)$.

[10] Suppose he chooses the $(2-1)$ scheme. Therefore in period 1 he offers a long term contract involving (α_1^h, β_1^h), (α_2^h, β_2^h) and a short term contract involving $(\alpha_1^\ell, \beta_1^\ell)$.

Stage 4. The tenant decides whether to reject both contracts on offer (in which case production does not take place in period 2) or to accept exactly one of them followed by his choice of effort and period 2 production. In either case, the game ends.

The payoffs of the players are as follows. Suppose the tenant being of type θ is working in period t under a contract $\left(\alpha_t^\theta, \beta_t^\theta\right)$. Then his period t payoff is

$$u\left(e_t; \alpha_t^\theta, \beta_t^\theta, \theta\right) = u\left(\left(1-\alpha_t^\theta\right)\theta f\left(e_t\right) - e_t - \beta_t^\theta\right). \tag{1}$$

We assume that the tenant is *risk averse*, meaning $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Also, we assume that for any $\alpha \in (0,1)$, there exits a level of effort e^* such that $(1-\alpha)\theta f(e^*) - e^* > 0$. That is, there exists an optimal level of effort that yields a positive payoff (before paying the rent) to the tenant even for a very small but positive share of the output. Furthermore, let $\pi > 0$ be the tenant's subsistence payoff which is the minimum he must receive in each of the two periods.

The landlord is *risk neutral*. If he offers (α_1, β_1) and (α_2, β_2) under either a short or a long term contract to the tenant who is of type θ, the landlord's payoff over the two periods is[11]

$$\Pi\left((\alpha_t, \beta_t)_{t=1,2}; e_t, \theta\right) = \sum_{t=1}^{2} \left(\alpha_t \theta f(\cdot) + \beta_t\right). \tag{2}$$

We look for a *perfect Bayesian equilibrium* of the above game. A similar model of asymmetric information in a static environment can be found in Hallagan (1978), Muthoo (1998) and Ray (1998, pp. 474-478). The rest of this section is devoted to the case where the skill of the tenant is known to the landlord.

2.2 First-Best Contracts

Consider the benchmark case where the skill of the tenant is common knowledge between the tenant and the landlord. For any given θ, the landlord has the option of offering short term or long term

[11] For simplicity we do not discount future payoffs. Any common strictly positive discount factor will not affect our results qualitatively.

contracts. However, it is easy to see in this case that the landlord is indifferent between the two durations. In any period t, the landlord offers $\left(\alpha_t^\theta, \beta_t^\theta\right)$ to the tenant of skill θ in order to solve

$$\begin{aligned} & \max_{\left(\alpha_t^\theta, \beta_t^\theta\right)} \alpha_t^\theta \theta f\left(e_t^\theta\right) + \beta_t^\theta \\ \text{subject to} \quad : \quad & u\left(e_t^\theta\left(\alpha_t^\theta, \beta_t^\theta\right); \theta, \alpha_t^\theta, \beta_t^\theta\right) \geq u(\pi). \end{aligned} \tag{3}$$

Given any contract $\left(\alpha_t^\theta, \beta_t^\theta\right)$, the tenant in period t chooses e_t^θ to solve

$$\max_{e_t^\theta} u\left(\left(1-\alpha_t^\theta\right)\theta f\left(e_t^\theta\right) - e_t^\theta - \beta_t^\theta\right). \tag{4}$$

The first order condition of the tenant's problem is

$$\theta f'\left(\widehat{e}_t^\theta\right) - 1 = \alpha_t^\theta \theta f'\left(\widehat{e}_t^\theta\right), \tag{5}$$

where $\widehat{e}_t^\theta$ is the optimum effort level chosen by the tenant with skill θ. Typically, $\widehat{e}_t^\theta$ will depend on the offered contract $\left(\alpha_t^\theta, \beta_t^\theta\right)$.

Since the landlord knows this and extracts all the surplus from the tenant, his maximization problem reduces to

$$\max_{\alpha_t^\theta} \theta f\left(\widehat{e}_t^\theta\right) - \widehat{e}_t^\theta - \pi.$$

The first order condition of the landlord's problem is therefore,

$$\left[\theta f'\left(\widehat{e}_t^\theta\right) - 1\right] \frac{d\widehat{e}_t^\theta}{d\alpha_t} = 0. \tag{6}$$

Since $\frac{d\widehat{e}_t^\theta}{d\alpha_t} \neq 0$, it follows from Eq.(6) that $\theta f'\left(\widehat{e}_t^\theta\right) - 1 = 0$. Using Eq.(5), this in turn implies that $\alpha_t^\theta = 0$, for any θ and t.

This is a standard result in the literature on tenancy contracts, and in our environment has the implication that in the event the skill of the tenant is known, the landlord is indifferent between hiring him for a short or a long period of time, and in both cases offers a fixed rent contract in either period. In the following sections, we will study optimal screening contracts that the landlord will offer under various informational structures, and concentrate on the issue of how the landlord can use the duration of such contracts to his own benefit.

3 Unknown skill but observable effort: *Adverse Selection*

So far we have studied the case where the landlord could identify the skill of the tenant he was hiring. In that case, the landlord is indifferent between a long term and a short term contract. Obviously, this benchmark model does not have much predictive power as it does not clearly justify one duration of contract over another. It is easy to see that if we had allowed for some uncertainty in the period 2 reservation payoff, the landlord would have preferred a long term contract for both types of tenants. Such a result is intuitively clear and in any case does not justify co-existence of different durations.

In this section we assume that the true skill of the tenant is not observed by the landlord and is held as private information by the tenant. Let us begin by analyzing the case where output and effort are both observable. In such a scenario, the tenant has no incentive to misrepresent his true skill as with full observability of output and effort the true skill of the tenant becomes common knowledge at the end of period 1 under any circumstance.

Two concepts regarding the notion of incentive compatibility mechanisms are required. One is the standard *period-incentive compatibility* constraint (PIC) which states that in any given period, a tenant weakly prefers the contract designed for his type over the contract designed for the other type. The second is the *intertemporal-incentive compatibility* constraint (IIC) which states that a tenant weakly prefers the contracts designed for his type over the two periods to the contracts designed for the other type. Observe that with only short term contracts, the two concepts are equivalent. However once we allow for the possibility of long term contracts, this equivalence may not always hold and the relevant constraint is the IIC. The concepts of separation and individual rationality are defined below.

Definition 1 ***Intertemporal Separation (IS)***

A $(t_h - t_\ell)$ scheme satisfies the property of IS if it satisfies the property of IIC.

Definition 2 ***Individual Rationality (IR)***

A $(t_h - t_\ell)$ scheme satisfies the property of IR if the tenant receives at least $u(\pi)$ in each period.

In the rest of the section we look for an optimal individually rational (IR) scheme which satisfies intertemporal separation (IS).

3.1 $(1-1)$ Scheme

Suppose the landlord wants to offer short term contracts to both types of tenants and separate them. In period 2, since the skill of the tenant becomes common knowledge, he knows that no matter what the true skill is, the contracts will entail fixed rents which extract the entire surplus from both types of tenants, as shown in section 2.2. In this respect, it is easy to see that $\left(\alpha_2^\ell, \beta_2^\ell\right) = \left(0, \beta_2^\ell\right)$ and $\left(\alpha_2^h, \beta_2^h\right) = \left(0, \beta_2^h\right)$, as depicted in figure (2). Consequently, the tenant is offered a single contract in period 2, implying that the only relevant incentive compatibility constraint is the PIC in period 1. Given this, the landlord solves the following problem to decide upon his separating period 1 contracts.

$$\max_{\left(\alpha_1^\ell, \beta_1^\ell\right), \left(\alpha_1^h, \beta_1^h\right)} \mu\left[\alpha_1^h h f\left(e_1^h\left(\alpha_1^h, \beta_1^h\right)\right) + \beta_1^h\right] + (1-\mu)\left[\alpha_1^\ell \ell f\left(e_1^\ell\left(\alpha_1^\ell, \beta_1^\ell\right)\right) + \beta_1^\ell\right] \tag{7}$$

subject to

$$\begin{array}{rcl} (i)\;\; u\left(e_1^h\left(\alpha_1^h, \beta_1^h\right); h, \alpha_1^h, \beta_1^h\right) & \geq & u(\pi) \\ (ii)\;\; u\left(e_1^\ell\left(\alpha_1^\ell, \beta_1^\ell\right); \ell, \alpha_1^\ell, \beta_1^\ell\right) & \geq & u(\pi) \end{array}$$

$$\begin{array}{rcl} (iii)\;\; u\left(e_1^h\left(\alpha_1^h, \beta_1^h\right); h, \alpha_1^h, \beta_1^h\right) & \geq & u\left(e_1^h\left(\alpha_1^\ell, \beta_1^\ell\right); h, \alpha_1^\ell, \beta_1^\ell\right) \\ (iv)\;\; u\left(e_1^\ell\left(\alpha_1^\ell, \beta_1^\ell\right); \ell, \alpha_1^\ell, \beta_1^\ell\right) & \geq & u\left(e_1^\ell\left(\alpha_1^h, \beta_1^h\right); \ell, \alpha_1^h, \beta_1^h\right), \end{array}$$

where $e_t^\theta(\alpha_t^{\theta'}, \beta_t^{\theta'})$ is the effort chosen by the tenant of skill θ while working in period t under the contract $(\alpha_t^{\theta'}, \beta_t^{\theta'})$ designed for the tenant of skill θ'.

In the above maximization problem, (i) and (ii) are the IR constraints, while (iii) and (iv) are the period 1 PIC constraints of the two types of tenants. The following lemma establishes some standard properties of the screening environment studied.[12]

[12] See Mas Collel et al. (1995), ch. 14, appendix B; and Ray (1998), ch. 12, appendix 2. We provide the proof

Lemma 3 *In period 1, the following is true.*

(i) The single-crossing property is satisfied in the (α, β) *plane.*

(ii) The high-skilled tenant always enjoys a positive surplus; the low-skilled tenant always gets zero surplus.

(iii) The high-skilled tenant gets a fixed-rent contract.

(iv) The period-incentive compatibility constraint of the low-skilled tenant is never binding, while that of the high-skilled tenant is binding.

(v) The low-skilled tenant gets a share contract.

Proof. See Appendix 7.1. ■

A typical menu of contracts under the $(1-1)$ scheme offered in equilibrium is depicted in fig.(1).

.... insert fig. (1) about here ...... [13]

In figure (1), the two short term contracts accepted by the high skilled tenant are $\left(0, \beta_1^h\right)$in period 1 and $\left(0, \beta_2^h\right)$ in period 2. Similarly, the two short term contracts accepted by the low skilled tenant are $\left(\alpha_1^\ell, \beta_1^\ell\right)$ in period 1 and $\left(0, \beta_2^\ell\right)$ in period 2.

An interesting observation at this stage is that under a $(1-1)$ scheme, the payoff of the low-skilled tenant is exactly equal to $u(\pi)$, the reservation payoff in both periods. This implies that although the low-skilled tenant remains as poor as he was without working in this farm, his payoff over the two periods is smoothed. However, due to the problem of separation, although in period 1 the high-skilled tenant enjoys a positive surplus, and feels wealthier, his payoff drops to his reservation payoff in period 2. Consequently, the high-skilled tenant experiences variations in his payoffs over the two periods. This observation will turn out to be central in what follows.

in an appendix to remain as self contained as possible. It is worth mentioning here that we are only looking for a separating equilibrium. In general, a pooling equilibrium may beat the separating in terms of profits to the landlord. However, this may not be the case if skills are far apart as in this case there are clear benefits from separating the two types. Consequently, we have implicitly assumed throughout the paper that skills are sufficiently different.

[13] All figures are provided at the end.

3.2 $(2-1)$ Scheme

Consider now the case where the landlord offers a long term contract to the high-skilled tenant and a short term contract to the low-skilled one. Since offering a short term contract, as in the case under a $(1-1)$ scheme, implies variations in the payoffs of the high-skilled tenant over the two periods, the landlord may take advantage of this by smoothing the high-skilled tenant's payoff. By doing this, the high-skilled tenant can be made weakly better off, while the landlord is able to extract a higher surplus from him over the two periods. However, this may lead to additional IIC problems which were absent under a $(1-1)$ scheme. A $(2-1)$ scheme involving the menu of contracts (α_1^h, β_1^h),(α_2^h, β_2^h), $(\alpha_1^\ell, \beta_1^\ell)$, $(\alpha_2^\ell, \beta_2^\ell)$ is IIC if

$$\begin{aligned} & u\left(e_1^h\left(\alpha_1^h, \beta_1^h\right); h, \alpha_1^h, \beta_1^h\right) + u\left(e_2^h\left(\alpha_2^h, \beta_2^h\right); h, \alpha_2^h, \beta_2^h\right) \\ \geq \; & u\left(e_1^h\left(\alpha_1^\ell, \beta_1^\ell\right); h, \alpha_1^\ell, \beta_1^\ell\right) + u\left(e_2^h\left(\alpha_2^h, \beta_2^h\right); h, \alpha_2^h, \beta_2^h\right), \end{aligned}$$

and

$$\begin{aligned} & u\left(e_1^\ell\left(\alpha_1^\ell, \beta_1^\ell\right); \ell, \alpha_1^\ell, \beta_1^\ell\right) + u\left(e_2^\ell\left(\alpha_2^\ell, \beta_2^\ell\right); \ell, \alpha_2^\ell, \beta_2^\ell\right) \\ \geq \; & u\left(e_1^\ell\left(\alpha_1^h, \beta_1^h\right); \ell, \alpha_1^h, \beta_1^h\right) + u\left(e_2^\ell\left(\alpha_2^h, \beta_2^h\right); \ell, \alpha_2^h, \beta_2^h\right). \end{aligned}$$

Since under a $(2-1)$ scheme the landlord offers a short term contract for the low skilled tenant, if the high skilled one accepts this short term contract, at the end of period 1 when the contract is terminated, he works under the (α_2^h, β_2^h) in period 2. On the other hand, if the low skilled tenant chooses to work under the contract offered for the high skilled one, he in turn locks himself up for 2 periods and therefore in period 2 works under the contract (α_2^h, β_2^h).

In the following proposition, we show that the optimal contracts under a $(2-1)$ scheme yield strictly higher payoffs to the landlord than those under a $(1-1)$ scheme.

Proposition 4 *In any optimal separating scenario, the landlord strictly prefers the* $(2-1)$ *scheme to the* $(1-1)$ *scheme.*

Proof. The proof is by construction. Take the optimal menu of contracts under the $(1-1)$ scheme. Consider figure (1) where such contracts are the points

$$\left(0,\beta_1^h\right),\left(0,\beta_2^h\right),\ \left(\alpha_1^\ell,\beta_1^\ell\right),\text{ and }\left(0,\beta_2^\ell\right).$$

Construct the following menu of contracts under a $(2-1)$ scheme.

$$\left(\alpha_1^h,\beta_1^h\right)=\left(\alpha_2^h,\beta_2^h\right)=\left(0,\beta^h\right),\left(\alpha_1^\ell,\beta_1^\ell\right),\text{ and }\left(0,\beta_2^\ell\right),$$

where $(0,\beta^h)$ is the "certainty equivalent" of the two contracts $(0,\beta_1^h)$ and $(0,\beta_2^h)$, defined as

$$\begin{aligned}&u\left(e_1^h\left(0,\beta^h\right);h{,}0,\beta^h\right)+u\left(e_2^h\left(0,\beta^h\right);h{,}0,\beta^h\right)\\=\;&u\left(e_1^h\left(0,\beta_1^h\right);h,\,0,\beta_1^h\right)+u\left(e_2^h\left(0,\beta_2^h\right);h,\,0,\beta_2^h\right).\end{aligned}$$

Since $u(\cdot)$ is strictly concave, we have $2\beta^h>\beta_1^h+\beta_2^h$, and therefore the landlord's payoffs are strictly greater under the contracts in a $(2-1)$ scheme. What remains to be shown is that the constructed contracts under a $(2-1)$ scheme are IR and IIC for both types of tenants. IR is easily established since the contracts $(\alpha_1^\ell,\beta_1^\ell)$ and $(0,\beta_2^\ell)$ yield a payoff of $u(\pi)$ in each period to the low-skilled tenant, while the contract $(0,\beta^h)$ yields positive surplus in each period to the high-skilled tenant. IIC for the low-skilled tenant is established by observing that a deviation in period 1 to the contract $(0,\beta^h)$ yields a payoff less than $u(\pi)$, which by definition is subsistence, and therefore such a deviation becomes impossible. To show that the IIC constraint of the high-skilled tenant is satisfied, we proceed as follows. The high-skilled tenant is indifferent between the contracts $(\alpha_1^\ell,\beta_1^\ell)$ and $(0,\beta_1^h)$. Since

$$\begin{aligned}&u\left(e_1^h\left(0,\beta^h\right);h{,}0,\beta^h\right)+u\left(e_2^h\left(0,\beta^h\right);h{,}0,\beta^h\right)\\=\;&u\left(e_1^h\left(0,\beta_1^h\right);h,\,0,\beta_1^h\right)+u\left(e_2^h\left(0,\beta_2^h\right);h,\,0,\beta_2^h\right).\end{aligned}$$

we have

$$\begin{aligned}&u\left(e_1^h\left(0,\beta^h\right);h{,}0,\beta^h\right)+u\left(e_2^h\left(0,\beta^h\right);h{,}0,\beta^h\right)\\=\;&u\left(e_1^h\left(\alpha_1^\ell,\beta_1^\ell\right);h,\,\alpha_1^\ell,\beta_1^\ell\right)+u\left(e_2^h\left(0,\beta_2^h\right);h,\,0,\beta_2^h\right),\end{aligned}$$

making any such deviation on part of the high-skilled tenant unprofitable for him. ■

The landlord is strictly better off by smoothing the payoffs of the high-skilled tenant over the two periods. Moreover, the incentive structure of the $(2-1)$ scheme is equivalent to that of the $(1-1)$ scheme. Therefore, the landlord strictly prefers the $(2-1)$ scheme to the $(1-1)$ scheme. In the following section, we will show that offering a long term contract to the high-skilled tenant to take advantage of the strict concavity of the tenant's payoff function becomes less attractive for the landlord if the low-skilled tenant works for a long term contract as well.

Remark 1. The constructed menu of contracts offered under the $(2-1)$ scheme in the above proof may not be the best one among the possible menus under that scheme. All we show in the above proposition is that it beats the best $(1-1)$ scheme. Notice that one can also construct a menu consisting of only fixed rent contracts which satisfy IS under the $(2-1)$ scheme. However, if skills are far apart, [see footnote 12], the best $(2-1)$ scheme will be a menu of contracts involving a fixed rent long term contract and a sequence of short term contracts, the first being a share contract while the second being a fixed rent.

3.3 $(2-2)$ Scheme

Consider the situation where the landlord wishes to offer separating contracts and hire both types of tenants for two periods. A $(2-2)$ scheme involving the menu of contracts (α_1^h, β_1^h),(α_2^h, β_2^h), $(\alpha_1^\ell, \beta_1^\ell)$, $(\alpha_2^\ell, \beta_2^\ell)$ is IIC if

$$\begin{aligned} & u\left(e_1^h\left(\alpha_1^h, \beta_1^h\right); h, \alpha_1^h, \beta_1^h\right) + u\left(e_2^h\left(\alpha_2^h, \beta_2^h\right); h, \alpha_2^h, \beta_2^h\right) \\ \geq\ & u\left(e_1^h\left(\alpha_1^\ell, \beta_1^\ell\right); h, \alpha_1^\ell, \beta_1^\ell\right) + u\left(e_2^h\left(\alpha_2^\ell, \beta_2^\ell\right); h, \alpha_2^\ell, \beta_2^\ell\right), \end{aligned}$$

and

$$\begin{aligned} & u\left(e_1^\ell\left(\alpha_1^\ell, \beta_1^\ell\right); \ell, \alpha_1^\ell, \beta_1^\ell\right) + u\left(e_2^\ell\left(\alpha_2^\ell, \beta_2^\ell\right); \ell, \alpha_2^\ell, \beta_2^\ell\right) \\ \geq\ & u\left(e_1^\ell\left(\alpha_1^h, \beta_1^h\right); \ell, \alpha_1^h, \beta_1^h\right) + u\left(e_2^\ell\left(\alpha_2^h, \beta_2^h\right); \ell, \alpha_2^h, \beta_2^h\right). \end{aligned}$$

Notice that the only difference in the IIC constraints between the $(2-1)$ and the $(2-2)$ schemes is in the second term of the R.H.S. in the first inequality. This arises out of the fact that now if the high skilled tenant chooses to work under the contract offered for the low skilled one, he in turn locks himself up for 2 periods and therefore in period 2 works under the contract $\left(\alpha_2^\ell, \beta_2^\ell\right)$. We first prove the following lemma which states that there exists menus of contracts which satisfy IR and IIC under the $(2-2)$ scheme. This lemma also guarantees the existence of an optimal menu of contracts under the $(2-2)$ scheme which satisfies the IS property.

Lemma 5 *Consider the case where the landlord offers contracts under the* $(2-2)$ *scheme. Then there exists a menu of contracts which satisfies IR and IIC.*

Proof. See Appendix 7.2. ■

Remark 2. Notice that the best menu of contracts under the $(2-2)$ scheme must satisfy the property that both cannot be pure fixed-rent contracts. Under a $(2-2)$ scheme, unlike the $(2-1)$ scheme, pure fixed-rent menus will never be separating as the high skilled tenant will always prefer the lower fixed rent arrangement due to the possibility of locking himself up for two periods with this low rent, which was clearly not possible under the $(2-1)$ scheme.

In the following proposition, we show that the landlord would always be strictly better off under a $(2-1)$ scheme than under a $(2-2)$ scheme.

Proposition 6 *In any optimal separating scenario, the landlord strictly prefers the* $(2-1)$ *scheme to the* $(2-2)$ *scheme.*

Proof. Lemma 5 proved that the set of separating contracts under a $(2-2)$ scheme is non-empty. Take an optimal contract under that scheme. We will prove that the landlord becomes better off by offering the same contract with the modification that the contract offered for the low type is not long term but rather a sequence of short term contracts. This is actually a $(2-1)$ scheme. The landlord under this scheme has to forego a lower information rent to separate the two types than under the $(2-2)$ scheme where a deviation is more profitable for the high type since even though his true skill is revealed by the end of period 1, he is entitled to pay the low rent β_2^l (as shown in fig.(2)) that the low-skilled tenant would pay, since he accepted a long term contract. Finally, it is easy to see that there is no extra cost associated with the $(2-1)$ scheme we proposed.

■

The IIC constraint for the high-skilled tenant is more binding whenever he has the option of tying himself up into a long term contract designed for the low-skilled tenant. Furthermore, a long term contract offered to the low-skilled tenant provides no additional benefit to the landlord. Therefore, instead of foregoing a higher information rent to separate the two types of tenants, the landlord can eliminate the available "tying-up" option which is preferred by the high-skilled tenant, by simply offering a short term contract to the low-skilled one.[14] Having established this intuition, the next proposition follows immediately.

3.4 $(1-2)$ Scheme

Suppose now that the landlord wishes to screen between the two types of tenants by offering a short term contract for the high-skilled tenant and a long term contract for the low-skilled one. The IIC constraints in this case are similar to the ones under the $(2-1)$ scheme with reversing roles of the types of the tenant. Notice that by offering a long term contract to the low-skilled tenant, the landlord is not able to reap any gain, as in any event, the payoffs of the low-skilled tenant are smoothed over the two periods and equal exactly to his period reservation payoff. Moreover, the $(1-2)$ scheme has the same incentive problems as in the $(2-2)$ scheme mentioned in subsection 3.3. Hence, a $(1-1)$ scheme dominates the $(1-2)$ scheme. The following proposition states this fact, a proof of which is not required.

Proposition 7 *In any optimal separating scenario, the landlord prefers the* $(1-1)$ *scheme to the* $(1-2)$ *scheme.*

This ends our study of different schemes of contracts that the landlord may wish to offer in order to separate between the two possible types of tenants when effort is observable. We summarize our

[14]Proposition 6 holds even when the tenant is risk neutral.

findings in the following theorem.

Theorem 8 *Suppose output and effort are perfectly observable by the landlord. Then, in any separating scenario, at the beginning of period 1, the landlord offers a long term fixed rent contract and a short term share contract. The high skilled tenant chooses to work under the long term contract while the low skilled tenant works under the short term one.*

4 Unknown skill and unobservable effort: *Moral Hazard and Adverse Selection*

In the model we study, the skill of the tenant is not observable in period 1. The landlord gets to know the true skill of the tenant only at the end of period 1 by observing the output produced and the effort exerted therein. However, in many instances, *absentee landlordism* being a common one, it may be the case that monitoring the actual level of effort becomes prohibitively costly for the landlord. In all such situations, the tenant may *strategically produce suboptimally* (given his true skills) in period 1 to misinform the landlord about his true skills, if doing so is beneficial for him. Since effort is not observable, such a misrepresentation is now feasible. For example, the low-skilled tenant may choose to put in more effort in order to ensure an output which only the high-skilled tenant would optimally produce. Similarly, the high-skilled tenant may choose to reduce his effort to produce what only the low-skilled tenant would optimally produce. Since the landlord does not have the time and resources to monitor effort, the contracts studied in section 3 may therefore not be intertemporally separating if such misrepresentations are allowed for.

Since the landlord is now aware of the fact that the tenant may or may not misrepresent his true skill, his separating scheme must now satisfy IIC under misrepresentation. A $(t_h - t_\ell)$ scheme is *IIC with misrepresentation* (IICM) if

$$\begin{aligned} & u\left(e_1^h\left(\alpha_1^h, \beta_1^h\right); h, \alpha_1^h, \beta_1^h\right) + u\left(e_2^h\left(\alpha_2^h, \beta_2^h\right); h, \alpha_2^h, \beta_2^h\right) \\ \geq\ & u\left(e_1^\ell\left(\alpha_1^\ell, \beta_1^\ell\right); h, \alpha_1^\ell, \beta_1^\ell\right) + u\left(e_2^h\left(\alpha_2^\ell, \beta_2^\ell\right); h, \alpha_2^\ell, \beta_2^\ell\right), \end{aligned}$$

and

$$\begin{aligned} & u\left(e_1^\ell\left(\alpha_1^\ell, \beta_1^\ell\right); \ell, \alpha_1^\ell, \beta_1^\ell\right) + u\left(e_2^\ell\left(\alpha_2^\ell, \beta_2^\ell\right); \ell, \alpha_2^\ell, \beta_2^\ell\right) \\ \geq\ & u\left(e_1^h\left(\alpha_1^h, \beta_1^h\right); \ell, \alpha_1^h, \beta_1^h\right) + u\left(e_2^\ell\left(\alpha_2^h, \beta_2^h\right); \ell, \alpha_2^h, \beta_2^h\right), \end{aligned}$$

where now, $e_t^\theta\left(\alpha_t^{\theta'},\beta_t^{\theta'}\right)$ is the effort chosen by the tenant of skill θ while working in period t under the contract $(\alpha_t^{\theta'},\beta_t^{\theta'})$ designed for the tenant of skill θ' when misrepresentation is feasible. The first inequality implies that the high skilled tenant is weakly better off by working under the contracts designed for him than working under the period 1 contract designed for the low skilled tenant, misrepresenting in period 1, and then working in period 2 under the contract designed for the low skilled tenant and working according to his true skills. Clearly, *no type of tenant would like to misrepresent in period 2*, as misrepresentation in any given period is costly for that period, and yields no benefit if there is no future! Similar conditions are represented in the second inequality for the low skilled tenant. Notice that the IR conditions will remain as in section 3. The following lemma will be essential for theorem 10. It shows that under a $(1-1)$ scheme, there exists a menu of contracts over the two periods such that IR and IICM are satisfied.

Lemma 9 *Consider the case where the landlord offers contracts under the* $(1-1)$ *scheme. Then, there exists a menu of contracts which satisfies IR and IICM.*

Proof. See Appendix 7.3. ■

The above lemma asserts that the set of separating $(1-1)$ contracts is non-empty. This fact is used in the next theorem where we compare again the four schemes under the assumption that effort is not observable. In the following theorem, we show that even when the landlord has to take additional care regarding this possibility of misrepresentation, theorem 8 will hold, at least qualitatively. That is, although the exact contracts offered may change, the landlord's preference over their durations will not.

Theorem 10 *Suppose output is observable but* ***effort is not****. Then in any separating scenario, at the beginning of period 1 the landlord offers a long term fixed rent contract and a short term share contract. The high skilled tenant chooses to work under the long term contract while the low skilled tenant works under the short term one.*

Proof. Suppose it is common knowledge that the tenant can misrepresent his true skills. Notice that the low-skilled tenant does not misrepresent in any period. To see this, observe that in any separating scheme, the low-skilled tenant gets a zero surplus[15] and by definition, misrepresentation is costly due to the suboptimality of altering the optimal effort level. As mentioned in the proof of lemma 9, i) the high-skilled tenant may have incentives to misrepresent his true skills and ii) no tenant has any incentive to misrepresent his true skills in period 2 as misrepresentation is always costly and yields no extra benefit in the terminal period.

We proceed as follows.

Step 1. We begin by comparing the $(1-1)$ scheme with the $(2-1)$ scheme. Let the contracts under the $(1-1)$ scheme in this case be

$$\left(\widehat{\alpha}_1^h, \widehat{\beta}_1^h\right), \ \left(\widehat{\alpha}_2^h, \widehat{\beta}_2^h\right), \left(\widehat{\alpha}_1^\ell, \widehat{\beta}_1^\ell\right), \ \left(\widehat{\alpha}_2^\ell, \widehat{\beta}_2^\ell\right).$$

By lemma 3 it follows that $\widehat{\alpha}_1^h = \widehat{\alpha}_2^h = 0$, $\widehat{\alpha}_1^\ell \in (0,1)$ and $\widehat{\alpha}_2^\ell = 0$.

Suppose the landlord offers the same contracts under a $(2-1)$ scheme with the following alterations. The contracts offered for the high skilled tenant are $\left(0, \beta^h\right)$ in each period, where the contract $\left(0, \beta^h\right)$ is the "certainty equivalence" of the two contracts $\left(0, \widehat{\beta}_1^h\right)$ and $\left(0, \widehat{\beta}_2^h\right)$ offered under the $(1-1)$ scheme. Thus, we have

$$\begin{aligned} & u\left(e_1^h\left(0, \beta^h\right); h, 0, \beta^h\right) + u\left(e_2^h\left(0, \beta^h\right); h, 0, \beta^h\right) \\ = \ & u\left(e_1^h\left(0, \widehat{\beta}_1^h\right); h, 0, \widehat{\beta}_1^h\right) + u\left(e_2^h\left(0, \widehat{\beta}_2^h\right); h, 0, \widehat{\beta}_2^h\right). \end{aligned} \tag{9}$$

Define $G(1-1)$ and $G(2-1)$ to be the high-skilled tenant's gains from deviation and misrepresentation under the $(1-1)$ and the $(2-1)$ schemes respectively. Then,

$$\begin{aligned} G(1-1) = \ & \left\{u\left(e_1^\ell\left(\widehat{\alpha}_1^\ell, \widehat{\beta}_1^\ell\right); h, \widehat{\alpha}_1^\ell, \widehat{\beta}_1^\ell\right) + u\left(e_2^h\left(0, \widehat{\beta}_2^\ell\right); h, 0, \widehat{\beta}_2^\ell\right)\right\} \\ & - \left\{u\left(e_1^h\left(0, \widehat{\beta}_1^h\right); h, 0, \widehat{\beta}_1^h\right) + u\left(e_2^h\left(0, \widehat{\beta}_2^h\right); h, 0, \widehat{\beta}_2^h\right)\right\}, \end{aligned}$$

and

$$\begin{aligned} G(2-1) = \ & \left\{u\left(e_1^\ell\left(\widehat{\alpha}_1^\ell, \widehat{\beta}_1^\ell\right); h, \widehat{\alpha}_1^\ell, \widehat{\beta}_1^\ell\right) + u\left(e_2^h\left(0, \widehat{\beta}_2^\ell\right); h, 0, \widehat{\beta}_2^\ell\right)\right\} \\ & - \left\{u\left(e_1^h\left(0, \beta^h\right); h, 0, \beta^h\right) + u\left(e_2^h\left(0, \beta^h\right); h, 0, \beta^h\right)\right\}. \end{aligned}$$

The first two terms on the R.H.S. of the above two equations denote the payoff to the high-skilled tenant working under the contracts designed for the low-skilled tenant when he misrepresents,

[15] The proof of this is similar to that of lemma 3, part (ii).

while the remaining terms denote the payoff to the high-skilled tenant working (according to his true skill) under the contracts designed for his type. Clearly, by construction, in equilibrium we have $G(1-1) = G(2-1) \leq 0$. Thus, an optimal $(1-1)$ contract is also IS under the $(2-1)$ scheme. Thus, by Eq.(9) and the fact that $u(\cdot)$ is strictly concave, $2\beta^h > \beta_1^h + \beta_2^h$, and the landlord prefers the constructed $(2-1)$ scheme to the optimal $(1-1)$ scheme. The best $(2-1)$ scheme therefore clearly yields a strictly greater payoff to the landlord than the optimal $(1-1)$ scheme. This shows that the landlord strictly prefers the $(2-1)$ scheme to the $(1-1)$ scheme [Remark 1 applies here as well].

Step 2. Firstly, existence of a menu of contracts under the $(2-2)$ scheme satisfying the IS property is easy to show by invoking lemma 5 [remark 2 applies here as well]. This can be seen easily since in any $(2-2)$ scheme whether effort is observable or not the tenant will not misrepresent. It is also easy to see that the landlord strictly prefers the $(2-1)$ scheme to the $(2-2)$ scheme as well. The argument is similar to the proof of proposition 6 and therefore we omit it here.

Step 3. Finally, the landlord strictly prefers the $(1-1)$ scheme to the $(1-2)$ scheme. The argument is exactly the same as in the comparison between the $(2-1)$ scheme and the $(2-2)$ scheme.

From Steps 1-3, it then follows that the landlord strictly prefers the $(2-1)$ scheme to any other scheme of contracting. ■

Whether effort is observable or not, we see that in any optimal screening scenario where the landlord wishes to separate between the high and the low skilled tenants, he typically offers a long term contract to select the high-skilled tenant and a short term contract to select the low-skilled one. The driving forces behind this general result are the following. Firstly, in any separating outcome where the high-skilled tenant works under a short term contract, there is a variation in his payoffs over the two periods. Therefore, offering a contract with a longer duration enables the landlord to extract a greater lifetime surplus from the high-skilled tenant by extracting the extra payoff that the tenant enjoys from the smoothing of his payoff stream. However, the landlord needs to be careful of another aspect. Since the contract offered in period 2 to the low-skilled tenant is attractive to the high skilled one, he must make sure that the high-skilled tenant does not accept the long term contract designed for the low-skilled one. Offering only a short term contract to the low-skilled tenant does this job and entails a lot less information rent on part of the landlord as against the case where he separates by offering long term contracts to both types of tenants. Also, offering a short term contract to the low-skilled tenant does not reduce the payoff of the landlord as the low-skilled tenant's payoff is smoothed under any separating contractual scheme. Of course, with the possibility of misrepresentation, the overall information rent foregone by the landlord may be higher than when such misrepresentations are ruled out. Nevertheless, the preference ordering of the landlord over different contractual durations for different skills remains unaffected.

5 More than 2 periods: an informal discussion

So far, we have assumed that the planning horizon comprises of only two periods. In doing so we have been able to provide a theoretical explanation to Newbery's claim that fixed rent contracts are of longer duration than sharecropping arrangements. However, while we have been able to capture the basic spirit of the issue regarding duration of tenancy contracts, the model we have used so far is partially unable to explain the exact findings of Bandiera from 19th Century Sicily where durations of one, two, three, four and more years are observed. In this light, one may wonder if the established preference of the landlord over the duration of contracts is robust to many periods. In this section we address this issue in a fairly informal manner.

We begin by extending the two-period game form described in section 2.1 to n periods, $2 < n < \infty$. Whether we allow for effort to be observable or not, and therefore whether the tenant will have the possibility of misrepresenting his true skill or not, the basic argument will remain similar. For the sake of simplicity,[16] let us assume that effort is observable. Let $\Theta = \{1, ...k\}$ be the set of possible skills of the tenant, with $\theta \in \Theta$.

If $k = 2$, then we claim that the unique separating equilibrium of the $n-$period game form is as follows: *the landlord offers and $n-$ period long term fixed rent contract and a 1-period short term sharecropping contract at the beginning of period 1; a high skilled tenant works under the long term arrangement while a low skilled tenant works under the short term one.* To see this observe

[16] The argument when effort is not observable is more involved but shares the same spirit.

that when $k = 2$, the problem faced by the landlord is essentially the same with the 2-period game form studied in sections 3 and 4. In particular, under the optimal $(1-1)$ scheme over n periods, the payoff of the low skilled tenant remains fixed at $u(\pi)$ in each of the n periods while the high skilled tenant receives a surplus in period 1 and $u(\pi)$ in each of the remaining $n-1$ periods. Suppose this period 1 payoff is $x > u(\pi)$. With strict concavity of $u(\cdot)$, it follows that the landlord finds it optimal to smooth the payoff of the high skilled tenant over the entire $n-$period horizon by offering him a fixed payoff strictly less than $[x + (n-1)u(\pi)]/n$ in each period. To make such a long term contract optimal IIC, the landlord must offer a one-period contract as an alternative for the low skilled tenant. This is because offering a contract with longer duration to the low skilled tenant provides no additional benefit while at the same time makes the long term contract offered to the high skilled tenant less attractive for him and requires on part of the landlord to forego larger information rents.

The equilibrium becomes more interesting when $k > 2$. Figure 2 depicts the first period contracts in a typical separating equilibrium under the $(1-1-1)$ scheme with $k = 3$ (clearly the second period contracts are all fixed rent contracts and are not shown in the figure).

...... insert fig. 2 about here ......

In general, such an equilibrium under the $(1-.....-1)_{k-\text{ times}}$ scheme exhibits the following features.

i) In the terminal $(n-1)$ periods, the contracts offered are fixed rent contracts, extracting the surplus from all types of tenants, while in period 1, tenant of skill k receives a fixed rent contract, while all other types from 1 to $k-1$ receive share contracts.

ii) Furthermore, the share for the $\theta-$th type is greater than that of the $(\theta+1)-$th type.

iii) Consequently, the tenant with skill $\theta = 1$ will earn zero surplus, while all tenants with skill 2 to k earn a positive surplus, with the surplus earned by the $\theta-$th type being more than that of the $(\theta+1)-$th type.

It is easy to see that the above $(1-...-1)_{k-\text{ times}}$ scheme is strictly less preferred by the landlord to the following $\left(n-(1-....-1)_{(k-1)-\text{ times}}\right)$ scheme: the landlord offers the certainty equivalent fixed rent contract of the $(1 - ... - 1)_{k-\text{ times}}$ scheme for $n-$ periods to the tenant of skill k, while all other contracts remain the same as above. Notice that under this new arrangement, there is still some payoff variation experienced by the tenants of type 2 to $k-1$. The question is can the landlord do better by smoothing the income of some other types of tenants? Clearly, his first choice would be to smooth the payoff of the tenant with skill $k-1$ since the $(k-1)-$th type experiences the highest payoff variation in this new setting. However, the landlord cannot offer an $n-$period certainty equivalent fixed rent contract to the $(k-1)-$th type since then the $k-$th type tenant will not work under the contract designed originally for him and grab the $n-$period one designed for the $(k-1)-$th type. The optimal choice of the duration of this certainty equivalent fixed rent contract for type $k-1$ in this setting is beyond the scope of this paper. However, in any IS equilibrium, *the highest type will receive a long term $n-$period fixed rent contract while the lowest type will receive a 1- period share contract. The intermediate types will receive contracts with durations ranging from 1 to $n-1$.* Our conjecture is that one possible equilibrium will exhibit the following ladder: *as the skill of the tenant decreases the length of the contract decreases as well.* However the exact nature of these contracts remains an open question.

6 Conclusion

A landlord can always offer contracts such that in each period he guarantees to pay according to the skill of the tenant even if such skills are not perfectly observable. These are the standard screening contracts studied extensively in economics. The question is, *can he do better*? In this paper we have shown that the answer is certainly yes if the tenants are more risk averse. The landlord strictly increases his lifetime payoffs by taking advantage of the fact that he can also differentiate between the contracts offered to the two types of tenants in the dimension of time. In particular, his best scheme is to offer long term contracts to high-skilled tenants and short term contracts to low-skilled tenants. It is then not surprising at all that short and long term contracts may co-exist as has been empirically verified by Bandiera. Also, Newbery's claim that share cropping is typically short term while fixed rent contracts are long term may also be supported by the theory we provide here. In this paper we have throughout assumed the case of full commitment. An interesting extension would be to study the robustness of our results in the face of renegotiation in each period. We reserve this for future research.

7 Appendix

7.1 Proof of Lemma 3

(i) Given the payoff function of the tenant, along any iso-payoff curve of the tenant of type θ, we have

$$\frac{d\beta}{d\alpha}|_{\theta} = -\theta f\left(e_1^{\theta}\left(\alpha_1, \beta_1\right)\right).$$

It is easy to see that for any contract (α_1, β_1), we have $e_1^h(\alpha_1, \beta_1) > e_1^{\ell}(\alpha_1, \beta_1)$. Since β does not affect the effort level, we have

$$\frac{d\beta}{d\alpha}|_h < \frac{d\beta}{d\alpha}|_{\ell} \qquad \text{for all } (\alpha_1, \beta_1) \in [0,1] \times \mathbb{R}.$$

This proves part (i).

(ii) From the individual rationality and the period-incentive compatibility constraints of the tenant, and the fact that for any given contract (α, β), the high-skilled tenant generates more payoff than the low-skilled tenant, we have

$$\begin{aligned} & u\left(e_1^h\left(\alpha_1^h, \beta_1^h\right); h,\, \alpha_1^h,\, \beta_1^h\right) \\ \geq\ & u\left(e_1^h\left(\alpha_1^{\ell}, \beta_1^{\ell}\right); h,\, \alpha_1^{\ell},\, \beta_1^{\ell}\right) \\ >\ & u\left(e_1^{\ell}\left(\alpha_1^{\ell}, \beta_1^{\ell}\right); \ell,\, \alpha_1^{\ell},\, \beta_1^{\ell}\right) \geq u(\pi). \end{aligned}$$

This proves that the high-skilled tenant always enjoys a positive surplus.

To prove that the low-skilled tenant does not get any surplus, we proceed as follows. Let $\left(\alpha_1^h, \beta_1^h\right)$ and $\left(\alpha_1^{\ell}, \beta_1^{\ell}\right)$ be any two contracts such that constraints $(i) - (iv)$ in problem (7) are satisfied, and suppose on the contrary, we have

$$u\left(e_1^{\ell}\left(\alpha_1^{\ell}, \beta_1^{\ell}\right); \ell,\, \alpha_1^{\ell}, \beta_1^{\ell}\right) > u(\pi).$$

Construct a new pair of contracts $\left(\alpha_1^h, \beta_1^h + \varepsilon\right)$ and $\left(\alpha_1^{\ell}, \beta_1^{\ell} + \varepsilon\right)$. Then, by the fact that the high-skilled tenant enjoys a positive surplus, there exists an $\varepsilon^* > 0$ such that this new pair of contracts also satisfy constraints $(i)-(iv)$ of problem (7), and the landlord clearly prefers it to the original contract. This contradicts with $\left(\alpha_1^h,\beta_1^h\right)$ and $\left(\alpha_1^\ell,\beta_1^\ell\right)$ being optimal.

(iii) Let $\left(\alpha_1^\ell,\beta_1^\ell\right)$ be the contract offered to the low-skilled tenant.

.... **insert fig. 3 about here** ....

Then, by the single crossing property as established in (i), the contract $\left(\alpha_1^h,\beta_1^h\right)$ offered to the high-skilled tenant must lie on the line segment $[x,y)$ as in fig.(3). We claim that $\left(\alpha_1^h,\beta_1^h\right)$ coincides with the point $x=\left(0,\beta_1^h\right)$. To prove this, it is sufficient to observe that if the landlord faces only the constraint of the high-skilled tenant, the optimal choice of $\left(\alpha_1^h,\beta_1^h\right)$ would be x, which is a fixed-rent contract.

(iv) This follows trivially from part (iii).

(v) This follows trivially from the separation property[17] of the problem and part (iii).

7.2 Proof of lemma 5

Consider fig. (4).

.... **insert fig. 4 about here** ......

Choose some ε_1, $\varepsilon_2>0$ and construct the menu of IR contracts under the $(2-2)$ scheme as

$$\mathcal{M}\left(\varepsilon_1,\varepsilon_2\right)=\left\{\left(0,\beta_2^\ell+\varepsilon_1\right),\ \left(1-\varepsilon_2,\beta_1^\ell\left(\varepsilon_2\right)\right),\ \left(0,\beta_2^\ell+\varepsilon_1\right),\ \left(0,\beta_2^\ell\right)\right\}$$

where the first long term contract is the fixed rent contract $\left(0,\beta_2^\ell+\varepsilon_1\right)$ for the two periods and the second long term contract is the share contract $\left(1-\varepsilon_2,\beta_1^\ell\left(\varepsilon_2\right)\right)$ in period 1 and the fixed rent contract $\left(0,\beta_2^\ell\right)$ in period 2. Also, $\beta_1^\ell\left(\varepsilon_2\right)$ is such that the contract $\left(1-\varepsilon_2,\beta_1^\ell\left(\varepsilon_2\right)\right)$ satisfies

$$u\left(e_1^\ell\left(1-\varepsilon_2,\beta_1^\ell\left(\varepsilon_2\right)\right);\ \ell,1-\varepsilon_2,\beta_1^\ell\left(\varepsilon_2\right)\right)=u\left(\pi\right)\ \text{ for all } \varepsilon_2\geq 0.$$

[17] Notice that given the problem at hand, any tenant working under a contract with α very close to 1 will also choose an effort level very close to zero. This would in turn imply that total output produced would be negligible, and so the landlord will receive almost nothing in return from the tenant. However, the tenant will have to be paid at least his reservation payoff of π, which would force the landlord to pay a very high negative rent (a negative β). Clearly therefore, the payoff to the landlord from hiring any tenant under a contract with α close to one (which includes $\alpha=1$) will be negative. Why would a landlord offer such a contract even in a separating environment, no matter how small is the probability of the tenant to have low skills? He can always do better by offering only a fixed-rent contract to the high-skilled tenant and exclude the possibility of hiring the low-skilled one. But this becomes a triviality, and the only interesting case for our purpose is when the landlord actually finds it optimal to offer two contracts to separate between the two skills.

In what follows we suppress this dependence of β_1^ℓ on ε_2. Clearly by construction, the low skilled tenant always chooses the second long term arrangement. If the high skilled tenant is indifferent between the two long term contracts, we are done. So, suppose that the high skilled tenant strictly prefers the second contract. Then it must be the case that

$$\begin{aligned} & u\left(e_1^h\left(1-\varepsilon_2,\beta_1^\ell\right);h,1-\varepsilon_2,\beta_1^\ell\right)+u\left(e_2^h\left(0,\beta_2^\ell\right);h,0,\beta_2^\ell\right) \\ > \quad & u\left(e_1^h\left(0,\beta_2^\ell+\varepsilon_1\right);h,0,\beta_2^\ell+\varepsilon_1\right)+u\left(e_2^h\left(0,\beta_2^\ell+\varepsilon_1\right);h,0,\beta_2^\ell+\varepsilon_1\right). \end{aligned}$$

We will show that there exist ε_1 and ε_2 such that the above inequality gets reversed, guaranteeing that $IIC \cap IR \neq \emptyset$. On the other hand we have

$$u\left(e_1^h\left(0,\beta_2^\ell+\varepsilon_1\right);h,0,\beta_2^\ell+\varepsilon_1\right)\to u\left(e_2^h\left(0,\beta_2^\ell\right);h,0,\beta_2^\ell\right) \text{ as } \varepsilon_1\to 0,$$

and

$$\lim_{\varepsilon_1\to 0} u\left(e_1^h\left(0,\beta_2^\ell+\varepsilon_1\right);h,0,\beta_2^\ell+\varepsilon_1\right) > \lim_{\varepsilon_2\to 0} u\left(e_1^h\left(1-\varepsilon_2,\beta_1^\ell\right);h,1-\varepsilon_2,\beta_1^\ell\right).$$

The last inequality follows from the fact that the two types of tenants have indifference curves with different slopes. Passing to the arguments with continuity, there exist ε_1^*, $\varepsilon_2^*>0$ such that for all $\varepsilon_1\in[0,\varepsilon_1^*]$ and $\varepsilon_2\in[0,\varepsilon_2^*]$, we have

$$\begin{aligned} & u\left(e_1^h\left(1-\varepsilon_2,\beta_1^\ell\right);h,1-\varepsilon_2,\beta_1^\ell\right)+u\left(e_2^h\left(0,\beta_2^\ell\right);h,0,\beta_2^\ell\right) \\ \leq \quad & u\left(e_1^h\left(0,\beta_2^\ell+\varepsilon_1\right);h,0,\beta_2^\ell+\varepsilon_1\right)+u\left(e_2^h\left(0,\beta_2^\ell+\varepsilon_1\right);h,0,\beta_2^\ell+\varepsilon_1\right). \end{aligned}$$

Thus the set of constructed contracts $\{\mathcal{M}\left(\varepsilon_1,\varepsilon_2\right)|\varepsilon_1\in[0,\varepsilon_1^*] \text{ and } \varepsilon_2\in[0,\varepsilon_2^*]\}$ satisfies IIC as well.

7.3 Proof of lemma 9

Consider fig (5).

.... **insert fig. 5 about here** ....

Choose $\varepsilon_1,\varepsilon_2>0$ and construct the menu of IR contracts $\mathcal{M}\left(\varepsilon_1,\varepsilon_2\right)$ as

$$\mathcal{M}\left(\varepsilon_1,\varepsilon_2\right)=\left\{\left(0,\beta_2^\ell+\varepsilon_1\right),\ \left(1-\varepsilon_2,\beta_1^\ell\left(\varepsilon_2\right)\right),\ \left(0,\beta_2^h\right),\ \left(0,\beta_2^\ell\right)\right\},$$

with the interpretation that $\left(0, \beta_2^\ell + \varepsilon_1\right)$ and $\left(0, \beta_2^h\right)$ are the two consecutive short term contracts designed for the high skilled tenant and $\left(1-\varepsilon_2, \beta_1^\ell\left(\varepsilon_2\right)\right)$ and $\left(0, \beta_2^\ell\right)$ are the two consecutive short term contracts designed for the low skilled tenant. Also, $\beta_1^\ell\left(\varepsilon_2\right)$ is such that the contract $\left(1-\varepsilon_2, \beta_1^\ell\left(\varepsilon_2\right)\right)$ satisfies

$$u\left(e_1^\ell\left(1-\varepsilon_2, \beta_1^\ell\left(\varepsilon_2\right)\right);\ \ell, 1-\varepsilon_2, \beta_1^\ell\left(\varepsilon_2\right)\right) = u\left(\pi\right) \text{ for all } \varepsilon_2 \geq 0.$$

In what follows we suppress this dependence of β_1^ℓ on ε_2. Notice that misrepresentation of the true skills can only hurt the low skilled tenant in each period [since he receives just his subsistence income]. Furthermore as mentioned before, no type of tenant will misrepresent in period 2. Given this, the only case of possible misrepresentation is from the high skilled tenant in period 1. So suppose the high skilled tenant chooses to work in period 1 under the contract $\left(1-\varepsilon_2, \beta_1^\ell\right)$ and misrepresent. Then he gets a payoff of $u(e_1^\ell\left(1-\varepsilon_2, \beta_1^\ell\right); h,\ 1-\varepsilon_2, \beta_1^\ell)$ in period 1 and $u\left(e_2^h\left(0, \beta_2^\ell\right); h, 0, \beta_2^\ell\right)$ in period 2. If $u(e_1^\ell\left(1-\varepsilon_2, \beta_1^\ell\right); h,\ 1-\varepsilon_2, \beta_1^\ell) < u\left(\pi\right)$, the proof ends. So suppose $u\left(e_1^\ell\left(1-\varepsilon_2, \beta_1^\ell\right); h, 1-\varepsilon_2, \beta_1^\ell\right) \geq u\left(\pi\right)$. If on the other hand he works under the contracts $\left(0, \beta_2^\ell + \varepsilon_1\right)$ and $\left(0, \beta_2^h\right)$ designed for him, his payoffs are $u\left(e_1^h\left(0, \beta_2^\ell + \varepsilon_1\right); h, 0, \beta_2^\ell + \varepsilon_1\right)$ in period 1 and $u\left(e_2^h\left(0, \beta_2^h\right); h, 0, \beta_2^h\right)$ in period 2. Now, the constructed menu of contracts satisfies IICM if

$$\begin{aligned} & u\left(e_1^h\left(0, \beta_2^\ell + \varepsilon_1\right); h, 0, \beta_2^\ell + \varepsilon_1\right) + u\left(e_2^h\left(0, \beta_2^h\right); h, 0, \beta_2^h\right) \\ \geq\ & u\left(e_1^\ell\left(1-\varepsilon_2, \beta_1^\ell\right); h, 1-\varepsilon_2, \beta_1^\ell\right) + u\left(e_2^h\left(0, \beta_2^\ell\right); h, 0, \beta_2^\ell\right). \end{aligned} \tag{8}$$

It is easy to see that

$$u\left(e_1^h\left(0, \beta_2^\ell + \varepsilon_1\right); h, 0, \beta_2^\ell + \varepsilon_1\right) \to u\left(e_2^h\left(0, \beta_2^\ell\right); h, 0, \beta_2^\ell\right) \text{ as } \varepsilon_1 \to 0,$$

and

$$u\left(e_1^h\left(1-\varepsilon_2, \beta_1^\ell\right); h, 1-\varepsilon_2, \beta_1^\ell\right) \to u\left(e_2^h\left(0, \beta_2^h\right); h, 0, \beta_2^h\right) \text{ as } \varepsilon_2 \to 0.$$

However, since misrepresentation is costly, it then follows that

$$\lim_{\varepsilon_2 \to 0} u\left(e_1^\ell\left(1-\varepsilon_2, \beta_1^\ell\right); h, 1-\varepsilon_2, \beta_1^\ell\right) < u\left(e_2^h\left(0, \beta_2^h\right); h, 0, \beta_2^h\right).$$

By continuity of Eq.(8) in ε_1 and ε_2, there exists an ε_1^* and $\varepsilon_2^* > 0$ such that for all $\varepsilon_1 < \varepsilon_1^*$ and $\varepsilon_2 < \varepsilon_2^*$, the inequality as in Eq.(8) becomes strict. This implies that the set of menus $\{\mathcal{M}\left(\varepsilon_1, \varepsilon_2\right) | \varepsilon_1 \in [0, \varepsilon_1^*] \text{ and } \varepsilon_2 \in [0, \varepsilon_2^*]\}$ satisfy IICM.

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FIGURES

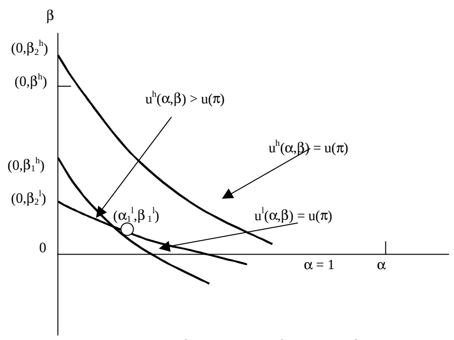

Figure 1. Optimal Contracts over the two periods

Figure 1:

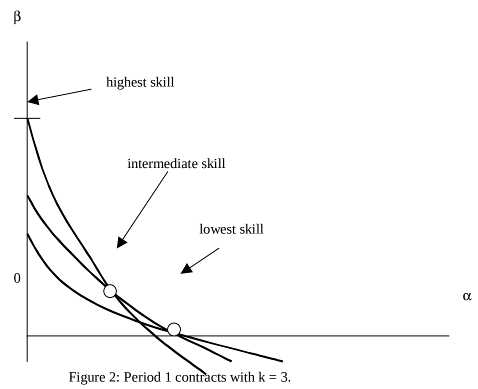

Figure 2: Period 1 contracts with k = 3.

Figure 2:

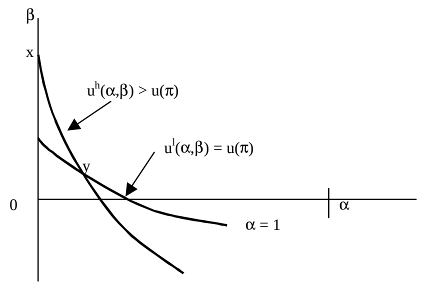

Figure 3: Nature of separating contracts.

Figure 3:

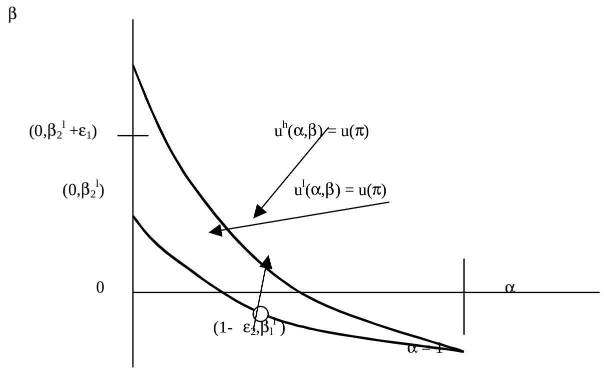

Figure 4: Existence of a (2-2) separating scheme.

Figure 4:

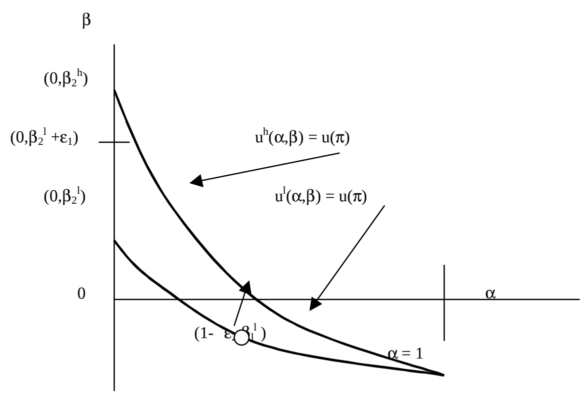

Figure 5: Existence of a (1-1) separating scheme under misrepresentation.

Figure 5: